MA 2550: Calculus I (Fall 2010)
Review for Final Exam

The Final Exam is cumulative, which means that any material that we have covered this semester is fair game. Questions on material covered on the first three exams will be similar in nature to the questions asked on those exams. In fact, you may see some of the same questions again. You are also responsible for the material covered since the last exam.

Note: You may bring one **8.5 inch by 11 inch cheat sheet** with you to the exam.

The Final Exam is worth 20% of your final grade and will take place on **Tuesday, December 14** at 11:00–1:30PM in Hyde 313.

Topics

To be successful on the material covered since Exam 3, you should

- be able to find antiderivatives and evaluate indefinite integrals
- be able to approximate the area (or net signed area) under the graph of a function using rectangles
- be able to approximate a definite integral of a function over an interval using rectangles
- be able to compute the exact value of a definite integral of a function over an interval by taking limits of Riemann sums (using right endpoints)*
- know statements of both parts of the Fundamental Theorem of Calculus
- be able to compute derivatives of functions defined in terms of integrals (using Part 1 of the Fundamental Theorem of Calculus)
- be able to compute definite integrals of functions using Part 2 of the Fundamental Theorem of Calculus
- be able to find net distance and total distance traveled by an object over an interval of time given a velocity function
- be able to find the area between two curves
- be able to evaluate indefinite and definite integrals using u-substitution

Words of advice

Here are somethings to keep in mind when taking the exam:

- Show all work! The thought process and your ability to show *how* and *why* you arrived at your answer is more important to me than the answer itself. For example, if you have the right answer, but your reasoning is flawed, then you will lose most of the points.
- The exam will be designed so that you could complete it without a graphing calculator. If you find yourself using your calculator a lot on a given question, then you may be doing something wrong.
- Make sure you have answered the question that you were asked. Also, ask yourself if your answer makes sense.

*I will provide you with the summation formulas and formulas for Δx and x_i^* when each x_i^* is the right endpoint of the ith subinterval.

- If you know you made a mistake, but you can't find it, explain to me why you think you made a mistake and tell me where the mistake might be. This shows that you have a good understanding of the problem.
- If you write down an "=" sign, then you better be sure that the two expressions on either side are equal. Similarly, if two things are equal and it is necessary that they be equal to make your conclusion, then you better use "=."
- Don't forget to write limits, integral symbols, $+C$, dx, etc. where they are needed.
- Both of us should be able to read what you wrote. Your work should be neat and organized! In general, your work should flow from left to right and then top to bottom (just like if you were reading). Don't make me wander around the page trying to follow your work.
- If your answer is not an entire paragraph (and sometimes it may be), then your answer should be clearly marked.
- Ask questions when you are confused. I will not give away answers, but if you are confused about the wording of a question or whether you have shown sufficient work, then ask me.

Exercises

Try some of these problems. There are a lot of problems below and you don't necessarily need to do all of them. You should do the ones that you think you need more practice on. I'm hoping that you will talk amongst each other to determine if you are doing them correctly. Of course, if you have questions, then I will answer them. Lastly, if a concept appears in multiple questions, you should not necessarily take that to mean that that concept is somehow more important than ones that do not appear frequently.

1. True or False? Justify your answer.
$$\int_{a/2}^{b/2} f(2x)\ dx = \frac{1}{2}\int_a^b f(x)\ dx$$
2. Explain the difference between an indefinite integral and a definite integral.
3. Explain why the Power Rule for Integration, $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C$, does not apply to the function $f(x) = \frac{1}{x}$.
4. Find f such that $f'(x) = \sqrt{x}$ and $f(4) = 0$.
5. Approximate the area under the graph of $f(x) = \cos^2 x$ on the interval $[0, \pi]$ by forming 4 rectangles and using right endpoints to determine the height of the rectangles.
6. Evaluate the following definite integrals by taking the limit of Riemann sums. (Use $x_i^* = x_i$; that is, use right endpoints.)
   (a) $\int_0^1 x^2\ dx$
   (b) $\int_{-1}^3 2x - 1\ dx$
7. Find $\int_1^{11} f(x)dx$ if $\int_0^1 f(x)dx = -7$ and $\int_0^{11} f(x)dx = 29$.

8. Draw a picture that represents the function $f(x) = \int_0^x \sin t^2 \, dt$ and then find its derivative.

9. Find the derivative of $g(x) = \int_1^{x^2} (t^2+1)^3 \, dt$.

10. Evaluate each of the following integrals.

    (a) $\int 4x^2 - 5x + 3 \, dx$

    (b) $\int \frac{\cos^3}{1-\sin^2 x} \, dx$ (*Hint:* use a trig identity)

    (c) $\int \frac{4+5x^{3/2}}{\sqrt{x}} \, dx$

    (d) $\int_{-2}^{5} 6 \, dx$

    (e) $\int_1^4 5 - 2x + 3x^2 \, dx$

    (f) $\int_0^{\pi} 3\sin x \, dx$

    (g) $\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta$

    (h) $\int 6 \, dx$

    (i) $\int x^5(5-2x+3x^2) \, dx$

    (j) $\int \frac{x^3+5}{x^2} \, dx$

    (k) $\int \frac{x^2}{\sqrt{x^3+5}} \, dx$

    (l) $\int_1^e \frac{\ln x}{x} \, dx$

    (m) $\int_0^{\pi/2} e^{\sin(x)} \cos(x) \, dx$

    (n) $\int \sin^3 x \cos x \, dx$

    (o) $\int \sec^2 x \tan x \, dx$

    (p) $\int \frac{1}{\sqrt{1-x}} \, dx$

    (q) $\int \frac{x}{\sqrt{1-x}} \, dx$

    (r) $\int_0^2 x\left(5-x^2\right)^{3/2} \, dx$

    (s) $\int_1^9 \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx$

    (t) $\int_0^{\pi} \sin x + \sin 3x \, dx$ (Be careful with your u-sub!)

(u) $\displaystyle\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$

11. Find the area of the region enclosed by the following graphs.

    (a) $y = 12 - x^2$ and $y = x^2 - 6$

    (b) $y = \dfrac{x^3 + 4}{x^2}$, $y = 0$, $x = 1$, and $x = 3$

    (c) $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \dfrac{\pi}{2}$

    (d) $y = x^3 - x^2 - 2x$ and $y = 4x$

12. Suppose $v(t) = -t^2 + 1$ is the velocity function for a nugget that moves in a straight line path.

    (a) Find the position function $p(t)$ if the nugget is at position $8/3$ when $t = 1$.

    (b) Describe the nugget's motion during the interval $[0, 2]$.

    (c) Using integration, find the net distance traveled by the nugget over the interval $[0, 2]$.

    (d) Using integration, find the total distance traveled by the nugget over the interval $[0, 2]$.

13. The following are two acceleration functions (with respect to time) for two different vehicles.

$$A_1(t) = \sqrt{t} \qquad \text{and} \qquad A_2(t) = \frac{1}{\sqrt{8}} t^2$$

    Find the area enclosed by the two acceleration curves and explain what it represents.

14. A common theme this semester has been to start with an approximation for something that is seemingly difficult to compute and then take a limit to get an exact answer. Describe ONE such situation that we have discussed this semester. I'm looking for an intuitive understanding, but you should provide some detail using proper notation. (Using pictures to aid in your description will be very useful.)

