Section 6.1: Optimization (part 2)

Goal

In this section, we will learn how to solve applied optimization (maximum and minimum) problems. In particular, we will take a look at word problems that deal with maximizing or minimizing a continuous function over a finite closed interval.

Guidelines for Solving Applied Optimization Problems

For every problem, we need to consider the *feasible domain* of the problem and then use techniques that we have already learned to maximize or minimize a particular function. Part of finding the solution to a given problem is making sure that we have actually found a solution.

Here are some steps to follow when attacking applied optimization problems:

1. Identify all given quantities and quantities to be determined. If possible, draw a picture.
2. Write a *primary equation* for the quantity that is to be maximized or minimized.
3. Reduce the primary equation to one having a single independent variable. This may involve the use of a *secondary equation* relating the independent variables of the primary equation.
4. Determine the *feasible domain* of the primary equation (this is some subset of the natural domain). That is, determine the values for which the stated problem makes sense. Often it is useful to include the endpoints despite the fact that something silly may happen there.
5. Determine desired max or min value by using the techniques of the previous sections.

Important Note 1. Sometimes we want to know what the max or min *is* and sometimes we want to know *when* the max or min occurs.

Examples

Let's do a few examples.

Example 2. A farmer has 500 feet of fencing with which to enclose a pasture for grazing nuggets. The farmer only need to enclose 3 sides of the pasture since the remaining side is bounded by a river (no, nuggets can't swim). In addition, some of the nuggets don't get along with some of the other nuggets. He plans to separate the troublesome nuggets by forming 2 adjacent corrals. Determine the dimensions that would yield the maximum area for the pasture.

Example 3. An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from each of the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

Example 4. An interstellar nugget travels along a path given by $p(x) = 4 - x^2$. If a star is located at the point $(0, 2)$, find the point(s) on the path of the nugget that are closest to the star.

If we have time, let's do this last one.

Example 5. Four feet of wire is used to form a square and a circle. How much wire should be used for the square and how much should be used for the circle to enclose the maximum combined area?